OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS

Core Mathematics 1

4721

QUESTION PAPER

Candidates answer on the Printed Answer Book

OCR Supplied Materials:
- Printed Answer Book 4721
- List of Formulae (MF1)

Other Materials Required:
None

Monday 24 May 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES
These instructions are the same on the Printed Answer Book and the Question Paper.

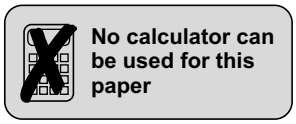
- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Printed Answer Book.
- **The questions are on the inserted Question Paper.**
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are **not** permitted to use a calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES
This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or destroyed.



1 **(i)** Evaluate 9^0. **[1]**

(ii) Express $9^{-\frac{1}{2}}$ as a fraction. **[2]**

2 **(i)** Sketch the curve $y = -\frac{1}{x^2}$. **[2]**

(ii) Sketch the curve $y = 3 - \frac{1}{x^2}$. **[2]**

(iii) The curve $y = -\frac{1}{x^2}$ is stretched parallel to the y-axis with scale factor 2. State the equation of the transformed curve. **[1]**

3 **(i)** Express $\frac{12}{3+\sqrt{5}}$ in the form $a - b\sqrt{5}$, where a and b are positive integers. **[3]**

(ii) Express $\sqrt{18} - \sqrt{2}$ in simplified surd form. **[2]**

4 **(i)** Expand $(x-2)^2(x+1)$, simplifying your answer. **[3]**

(ii) Sketch the curve $y = (x-2)^2(x+1)$, indicating the coordinates of all intercepts with the axes. **[3]**

5 Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$. **[5]**

6 Find the gradient of the curve $y = 2x + \frac{6}{\sqrt{x}}$ at the point where $x = 4$. **[5]**

7 Solve the simultaneous equations

$$x + 2y - 6 = 0, \qquad 2x^2 + y^2 = 57.$$ **[6]**

8 **(i)** Express $2x^2 + 5x$ in the form $2(x+p)^2 + q$. **[3]**

(ii) State the coordinates of the minimum point of the curve $y = 2x^2 + 5x$. **[2]**

(iii) State the equation of the normal to the curve at its minimum point. **[1]**

(iv) Solve the inequality $2x^2 + 5x > 0$. **[4]**

9 **(i)** The line joining the points A (4, 5) and B (p, q) has mid-point M (−1, 3). Find p and q. **[3]**

AB is the diameter of a circle.

(ii) Find the radius of the circle. **[2]**

(iii) Find the equation of the circle, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$. **[3]**

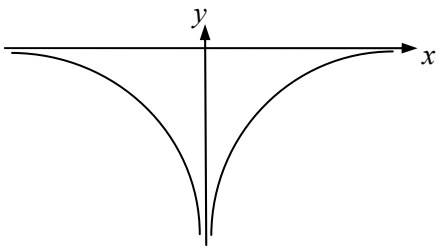
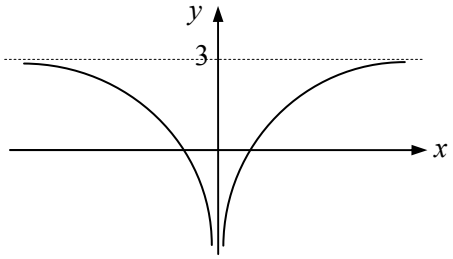
(iv) Find an equation of the tangent to the circle at the point (4, 5). **[5]**

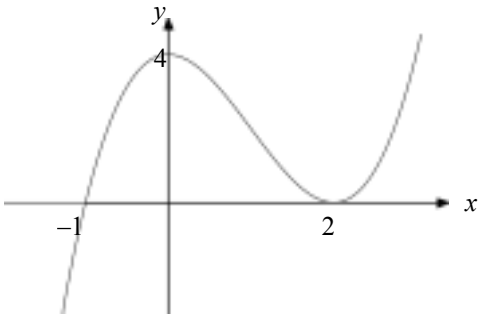
10 **(i)** Find the coordinates of the stationary points of the curve $y = 2x^3 + 5x^2 - 4x$. **[6]**

(ii) State the set of values for x for which $2x^3 + 5x^2 - 4x$ is a decreasing function. **[2]**

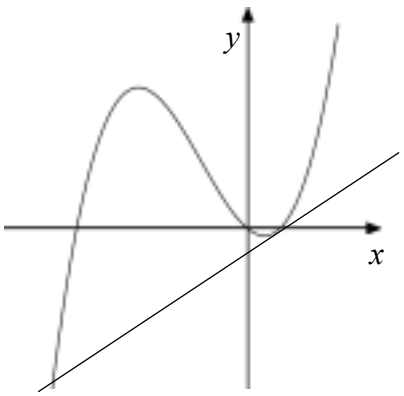
(iii) Show that the equation of the tangent to the curve at the point where $x = \frac{1}{2}$ is $10x - 4y - 7 = 0$. **[4]**

(iv) Hence, with the aid of a sketch, show that the equation $2x^3 + 5x^2 - 4x = \frac{5}{2}x - \frac{7}{4}$ has two distinct real roots. **[2]**

1 (i)	1	**B1**	**1**	
(ii)	$\frac{1}{3}$	**M1**		$\frac{1}{9^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{9}}$ soi
		A1	**2**	cao
			3	
2 (i)		**B1***		Reasonably correct curve for $y = -\frac{1}{x^2}$ in 3rd and 4th quadrants only
		B1 dep*	**2**	Very good curves in curve for $y = -\frac{1}{x^2}$ in 3rd and 4th quadrants **SC** If 0, very good single curve in either 3rd or 4th quadrant and nothing in other three quadrants. **B1**
(ii)		**M1**		Translation of their $y = -\frac{1}{x^2}$ vertically
		A1	**2**	Reasonably correct curve, horizontal asymptote soi at $y = 3$
(iii)	$y = -\frac{2}{x^2}$	**B1**	**1**	
			5	
3 (i)	$\frac{12(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$	**M1**		Multiply numerator and denom by $3-\sqrt{5}$
	$= \frac{12(3-\sqrt{5})}{9-5}$	**A1**		$(3+\sqrt{5})(3-\sqrt{5}) = 9-5$
	$= 9-3\sqrt{5}$	**A1**	**3**	
(ii)	$3\sqrt{2}-\sqrt{2}$	**M1**		Attempt to express $\sqrt{18}$ as $k\sqrt{2}$
	$= 2\sqrt{2}$	**A1**	**2**	
			5	

4 (i)	$(x^2-4x+4)(x+1)$	**M1**		Attempt to multiply a 3 term quadratic by a linear factor or to expand all 3 brackets with an appropriate number of terms (including an x^3 term)
		A1		Expansion with at most 1 incorrect term
	$=x^3-3x^2+4$	**A1**	**3**	Correct, simplified answer
(ii)		**B1**		+ve cubic with 2 or 3 roots
		B1		Intercept of curve labelled (0, 4) or indicated on *y*-axis
		B1	**3**	(–1, 0) and turning point at (2, 0) labelled or indicated on *x*-axis and no other *x* intercepts
			6	
5	$k=x^2$	**M1***		Use a substitution to obtain a quadratic or factorise into 2 brackets each containing x^2
	$4k^2+3k-1=0$			
	$(4k-1)(k+1)=0$	**M1 dep**		Correct method to solve a quadratic
	$k=\frac{1}{4}$ (or $k=-1$)	**A1**		
	$x=\pm\frac{1}{2}$	**M1**		Attempt to square root to obtain x
		A1	**5**	$\pm\frac{1}{2}$ and no other values
			5	
6	$y=2x+6x^{-\frac{1}{2}}$	**M1**		Attempt to differentiate
		A1		$kx^{-\frac{3}{2}}$
	$\frac{dy}{dx}=2-3x^{-\frac{3}{2}}$	**A1**		Completely correct expression (no +c)
	When $x=4$, gradient $=2-\frac{3}{\sqrt{4^3}}$	**M1**		Correct evaluation of either $4^{-\frac{3}{2}}$ or $4^{-\frac{1}{2}}$
	$=\frac{13}{8}$	**A1**	**5**	
			5	
7	$2(6-2y)^2+y^2=57$	**M1***		substitute for *x*/*y* or attempt to get an equation in 1 variable only
		A1		correct unsimplified expression
	$2(36-24y+4y^2)+y^2=57$			
	$9y^2-48y+15=0$	**A1**		obtain correct 3 term quadratic
	$3y^2-16y+5=0$			
	$(3y-1)(y-5)=0$	**M1 dep**		correct method to solve 3 term quadratic
	$y=\frac{1}{3}$ or $y=5$	**A1**		
	$x=\frac{16}{3}$ or $x=-4$	**A1**	**6**	**SC** If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**
			6	

8 (i)	$2(x^2+\frac{5}{2}x)$			
	$=2\left[\left(x+\frac{5}{4}\right)^2-\frac{25}{16}\right]$	**B1**		$\left(x+\frac{5}{4}\right)^2$
		M1		$q=-2p^2$
	$=2\left(x+\frac{5}{4}\right)^2-\frac{25}{8}$	**A1**	**3**	$q=-\frac{25}{8}$ c.w.o.
(ii)	$\left(-\frac{5}{4},-\frac{25}{8}\right)$	**B1√** **B1√**	**2**	
(iii)	$x=-\frac{5}{4}$	**B1**	**1**	
(iv)	$x(2x+5)>0$	**M1**		Correct method to find roots
		A1		0, $-\frac{5}{2}$ seen
	$x<-\frac{5}{2},\ x>0$	**M1**		Correct method to solve quadratic inequality.
		A1	**4** **10**	(not wrapped, strict inequalities, no 'and')
9 (i)	$\frac{4+p}{2}=-1,\quad \frac{5+q}{2}=3$	**M1**		Correct method (may be implied by one correct coordinate)
	p = -6 q = 1	**A1** **A1**	**3**	
(ii)	$r^2=(4-^-1)^2+(5-3)^2$	**M1**		Use of $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ for either radius or diameter
	$r=\sqrt{29}$	**A1**	**2**	
(iii)	$(x+1)^2+(y-3)^2=29$	**M1** **M1**		$(x+1)^2$ and $(y-3)^2$ seen $(x\pm1)^2+(y\pm3)^2=$ their r^2
	$x^2+y^2+2x-6y-19=0$	**A1**	**3**	Correct equation in correct form
(iv)	gradient of radius = $\frac{3-5}{-1-4}$	**M1**		uses $\frac{y_2-y_1}{x_2-x_1}$
	$=\frac{2}{5}$	**A1**		oe
	gradient of tangent = $-\frac{5}{2}$	**B1√**		oe
	$y-5=-\frac{5}{2}(x-4)$	**M1**		correct equation of straight line through (4, 5), any non-zero gradient
	$y=-\frac{5}{2}x+15$	**A1**	**5** **13**	oe 3 term equation e.g. $5x + 2y = 30$

10(i)	$\frac{dy}{dx} = 6x^2 + 10x - 4$	**B1**		1 term correct
		B1		Completely correct (no +c)
	$6x^2 + 10x - 4 = 0$	**M1***		Sets their $\frac{dy}{dx} = 0$
	$2(3x^2 + 5x - 2) = 0$			
	$(3x-1)(x+2) = 0$	**M1 dep***		Correct method to solve quadratic
	$x = \frac{1}{3}$ or $x = -2$	**A1**		**SC** If A0 A0, one correct pair of values, spotted or from correct factorisation **www** **B1**
	$y = -\frac{19}{27}$ or $y = 12$	**A1**	**6**	
(ii)	$-2 < x < \frac{1}{3}$	**M1**		Any inequality (or inequalities) involving both their x values from part (i)
		A1	**2**	Allow $\leq$ and $\geq$
(iii)	When $x = \frac{1}{2}$, $6x^2 + 10x - 4 = \frac{5}{2}$ and $2x^3 + 5x^2 - 4x = -\frac{1}{2}$	**M1**		Substitute $x = \frac{1}{2}$ into their $\frac{dy}{dx}$
		B1		Correct y coordinate
	$y + \frac{1}{2} = \frac{5}{2}\left(x - \frac{1}{2}\right)$	**M1**		Correct equation of straight line using their values. Must use their $\frac{dy}{dx}$ value not e.g. the negative reciprocal
	$10x - 4y - 7 = 0$	**A1**	**4**	Shows rearrangement to given equation **CWO** throughout for A1
(iv)		**B1**		Sketch of a cubic with a tangent which meets it at 2 points only
		B1	**2**	+ve cubic with max/min points and line with +ve gradient as tangent to the curve to the right of the min
			14	

SC1
B1 Convincing algebra to show that the cubic $8x^3 + 20x^2 - 26x + 7 = 0$ factorises into $(2x - 1)(2x - 1)(x + 7)$
B1 Correct argument to say there are 2 distinct roots
SC2 B1 Recognising $y = 2.5x - 7/4$ is tangent from part (iii)
B1 As second B1 on main scheme